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Accelerating Sphere—Wake Interaction

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Introduction

IT is the purpose of this Note to show briefly that an accelerating sphere in free fall (at intermediate Reynolds numbers) undergoes fluctuating lift (lateral) and drag forces that are directly related to an asymmetric vortex shedding phenomenon.

In a recent article,[1] Roos and Willmarth reported the drag and lift (lateral) forces acting on spheres. They observed that during the acceleration portion of the sphere's motion, there appears to be no relationship between the fluctuating drag and lift forces. From this evidence they quite reasonably suggest that "the first few cycles of vorticity shedding in the sphere wake must be axially symmetrical." This is very likely to be true in their case of an accelerating sphere constrained by a sting, since they observed that "the sting support had a significant stabilizing effect on the sphere wake." However, the present experiments show that in the absence of a sting support, the fluctuating lift during acceleration is directly connected to the asymmetric vortex shedding.

Experimental Apparatus

The experiments were conducted in a plexiglas water tower of 240-cm height and square cross section of 30 cm on each side. The lower half of the tank was reinforced by aluminum angles at the corners, held in place by bands of steel wire separated by a distance of 5 cm.

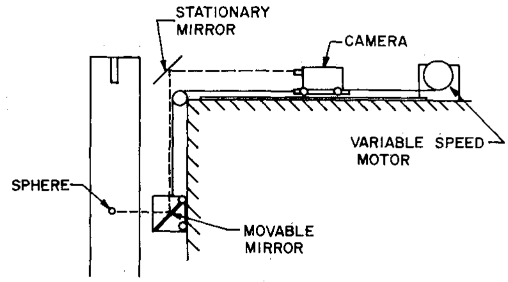

Fig. 1 Schematic of experimental apparatus.

Since the purpose of the experiment was to view the near wake of the sphere, it was decided to use spheres somewhat larger than those employed in previous free fall experiments.[2–4] The spheres chosen were table tennis balls filled with solids of varying densities ranging from wax through clay. The use of liquid-filled spheres was rejected since this would alter the dynamics of the system. One of the most successful methods of filling the spheres was to inject a gelatin solution into the sphere by the use of a hypodermic needle. The gelatin set, and the spheres thus produced had an over-all density somewhat greater than that of water and produced an intermediate Reynolds number motion.

A problem of some concern in freely falling sphere experiments in general and accelerating sphere experiments in particular is that of releasing the sphere with a minimum disturbance. Magnetic releasing mechanisms have been employed in the past but were of no help in this case since no metal spheres were to be used. Schafrir[4] used a vacuum system to hold the sphere. Such a system was attempted but the disturbance it caused was beyond the allowable limits for the acceleration investigation (although it appears to be sufficient for steady-state free fall experiments). The method chosen to release the spheres in the present experiment is a simple tube of inside diameter slightly greater than the outside diameter of the sphere. The spheres moved very slowly through the tube, taking on the order of 5 min to fall 20 cm and thus were released at sensibly zero velocity.

The data were recorded by photographing the spheres as they accelerated. Films were taken at 32 frames/sec as well as with a Nikon camera with a motor drive capable of taking 4 frames/sec. A tracking mechanism was constructed to allow the camera to follow the spheres accurately. A schematic of the tracker is shown in Fig. 1. The essential feature of the tracker is that the focal distance remains constant as the sphere falls. The camera rides on rails, its speed and direction being controlled by a variable speed aircraft motor con-

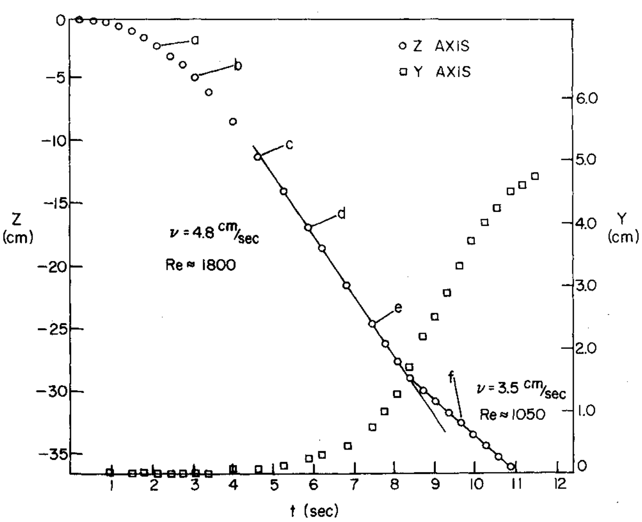

Fig. 2 Accelerating sphere trajectory.

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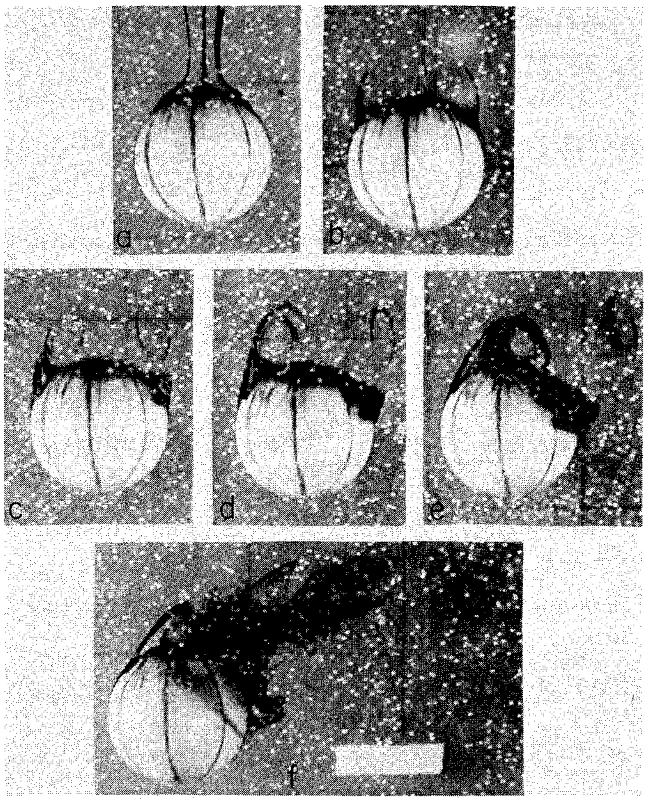

Fig. 3 Flow visualization of vortex shedding behind accelerating sphere.

nected to it by a cable. The other end of the camera platform is connected to another cable which passes over a pulley and is connected to a movable box containing a mirror inclined 45° to the horizontal. Therefore, as the camera moves to the right or left, the mirror box moves an equal distance up and down, respectively. The focal distance is the sum of the distance from the object to the movable mirror, the distance between mirrors and the distance between the stationary mirror and the camera. This total distance is a constant since the lateral motion of the object is essentially planar. Thus the camera may be set before the start of the experiment and requires no attention during the run. Full attention may be given to adjusting the motor speed accurately to track the accelerating sphere.

The flow visualization studies were pursued by coating the sphere with a concentrated solution of Methyl Blue Chloride as suggested in Ref. 5. It was found that by painting a line across the rear of the sphere such that the line lies in a plane normal to the line of sight of the camera, it was possible to obtain a cross-sectional view of the wake behind the sphere.

Experimental Results

Data were taken for Reynolds numbers (based on the constant velocity region following the initial acceleration) between 1×10^3 and 1×10^4. At the higher Reynolds numbers it became increasingly difficult to visualize the wake, although the dynamic behavior of the accelerating spheres remained unchanged.

A typical result for accelerating freely falling spheres is shown in Fig. 2. The Z axis indicates the distance that the sphere has fallen, whereas the Y axis indicates the deviation of the sphere's motion from a purely vertical trajectory. The photographs in Fig. 3 show the shape of the unsteady wake behind the sphere at selected points of the trajectory, as indicated in Fig. 2. The white spots are air bubbles that have formed on the tank walls. It may be seen from Fig. 2 that the sphere accelerates from zero velocity to constant velocity in approximately 5 sec. While the sphere is moving

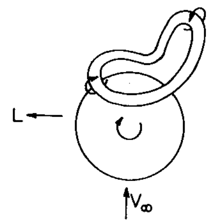

Fig. 4 Schematic of vortex shedding behind accelerating sphere.

at constant vertical velocity, it accelerates laterally until it attains a constant lateral velocity. The vertical velocity decreases rapidly at approximately $t = 8.5$ sec and assumes a new approximately constant velocity. The lateral velocity decreases more gradually.

The wake photographs in Fig. 3 clearly show that the wake is symmetrical during the acceleration portion of the trajectory (Figs. 3a-c). During the constant velocity portion of the trajectory, one side of the wake becomes unstable (Fig. 3d) and is shed (Fig. 3e). During this time the sphere itself moves toward the stable side of the wake. Finally (Fig. 3f), the entire wake has been shed and the vertical velocity of the sphere has decreased substantially. The lateral motion is also decreasing and the sphere is again approaching a purely vertical free fall condition.

A similar wake structure has been found in the case of a liquid sphere accelerating in a liquid medium.[6]

The experiments were repeated many times. It was not possible to predict which side of the wake would first become unstable. The releasing tube was positioned nearer to one side of the tank to determine if this would affect the sphere trajectory, but the direction of the initial wake shedding still appeared to be random.

Discussion and Conclusions

The experiments have shown that the lateral force on an unsupported accelerating sphere at intermediate Reynolds numbers is directly related to the unsteady shedding of the vortex wake. The physical mechanism proposed to clarify this phenomenon is shown in Fig. 4. A toroidally shaped wake is formed behind the sphere as it accelerates. This wake becomes unstable and a portion of it begins to be shed as depicted in Fig. 4. As this occurs, bound vorticity, of opposite sense to that being shed, is created on the sphere. The bound vorticity combined with the free fall velocity V_∞ leads to a lateral force L in the direction opposite to the initial shedding. This process is equivalent to the starting process in wing theory.

Another interesting aspect of the results in Fig. 2 is the rapid decrease of vertical velocity after the vortex wake has been shed. This probably occurs because the sphere must build a new recirculating wake to replace the one which was shed. To accomplish this, some fluid must be accelerated so that it travels with the sphere. The energy required is obtained from the kinetic energy of the falling sphere, and thus its vertical velocity decreases, corresponding to an increased drag.

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Tests of Cylindrical Shell under Concentrated Loads Applied to a Reinforcing Ring

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Introduction

ENGINEERS involved in the fields of submarines, missiles, aircraft, and pressure vessels use reinforced shells so extensively that investigations in this area are of continuing interest. For example, the stress analysis problem of a pressurized circular cylindrical shell reinforced by rings, one or more of which is externally loaded, is important in design considerations. In this Note, the authors provide experimental verification of the results from a previously obtained solution[1] of the closely related problem of a long circular cylindrical shell reinforced at its midlength by a single ring. Applied at the ends of a diameter of the ring are two self-equilibrating concentrated outward radial forces in the plane of the ring.

Curves showing stress distributions and displacements in the cylinder, obtained analytically and experimentally, are presented for a particular combination of ring and shell parameters.

Analytic Solution

The theoretical solution to the problem of an infinite shell, reinforced at its midlength by diametrically loaded rings, has been obtained by Allentuch, Brady, and Kempner.[1] In that solution they used the Donnell shell equations[2] with appropriate solutions given by Hoff[3] and Pohle and Nardo[4] to analyze the shell. The experiment under consideration in this Note was designed to verify some of the unusual results obtained in Ref. 1.

Experimental Setup

An experiment was designed to determine the stresses in and displacements of a long circular cylindrical shell reinforced at its midpoint by a single uniform ring. Applied to the opposite ends of a diameter of the ring were two equal concentrated outward radial loads.

The results obtained from the analytical solution[1] indicated that the maximum radial deflection of the shell did not occur at the ring. They further showed that the eccentricity e, between the ring and shell centerlines, had a first-order effect on the stresses in and displacements of the cylinder. These effects are not encountered in reinforced cylinders that deform axisymmetrically.

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Index categories: Submerged Vessel Systems, Structural Static Analysis.

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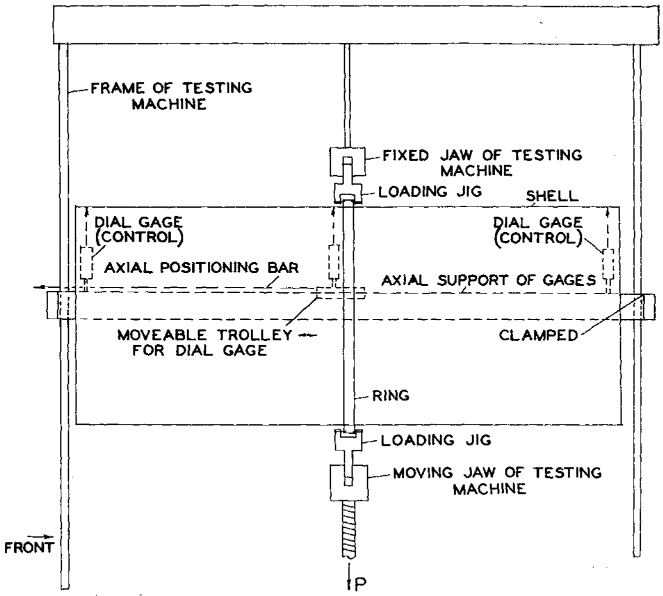

Fig. 1 Test setup (side view). Two end gages are control gages; center gage on moveable trolley is the gage used to measure the radial displacement.

In view of the unexpected behavior of some of the analytical results described previously, the experiment under consideration in this paper was designed. A 30-in. cylinder of mild steel was fabricated from sheet steel of nominal thickness 0.0125 in. An overlap seam was made by a line of spot welds. During the experiment, care was taken to keep this seam 90° from the applied diametral loads. The ring, a T-section, was also mild steel and was spot welded to the cylinder at its midlength.

The geometry of the combined structure is defined by the following: *ring*, $a_r \equiv$ centroidal radius $= 5.50$ in., $I \equiv$ centroidal moment of inertia of ring cross section $= 5.79136(10^{-3})$ in.^{4}, and $A \equiv$ area of ring cross section $= .211364$ in.^{2}; *shell*, $a \equiv$ mean radius of shell $= 5.00$ in., $I \equiv$ thickness of shell $= 0.0125$ in.

In the first experiment the radial deflection of the shell at different locations along a generator was obtained. Dial gages reading to 10^{-4} in. were employed to measure the radial displacement w.

The concentrated loads were applied to the T-section ring by a "jig" and pin arrangement at each end of a diameter (see Fig. 1). Roller bearings were used to eliminate friction so that the applied concentrated loads acted along a diameter. The tongues of the loading jigs were held in the jaws of the testing machine. The supports for the dial gages were rigidly attached to the frame of the testing machine.

Two different arrangements for the dial gages were used. In the first the gases were placed under the top load ($\varphi = \pi/2$). The experiment was then run with the gages at the bottom load position ($\varphi = 3\pi/2$).

Before a run was started, it was necessary to adjust the position of the loading jigs in the jaws of the testing machine so that during the application of the load no rigid body rotation of the shell axis would take place. To eliminate this rotation, control gages (see Fig. 1) were placed at each edge of the shell. Thus, by adjusting the position of the jig in the jaws of the testing machine so that the reading changes in the control gages were the same, a rigid body rotation of the cylinder was eliminated. It was found that once this adjustment was made, the positioning was relatively stable. How-